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## RESEARCH ARTICLE

## STABILITY ANALYSIS OF DC-DC BUCK CONVERTERS

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## ABSTRACT

This paper is focusing on the stability analysis of the voltage mode control buck converter controlled by pulse-width modulation (PWM). Using two different approaches, the nonlinear phenomena are investigated in two terms, slow scale and fast scale bifurcation. A complete design-oriented approach for studying the stability of dc-dc power converters and its bifurcation has been introduced. The voltage waveforms and attractors obtained from the circuit simulation have been studied. With the onset of instability, the phenomenon of subharmonics oscillations, quasi-periodicity, bifurcations, and chaos have been observed.

## KEYWORDS

DC-DC converters, PWM controller, bifurcation.

## 1. INTRODUCTION

DC-DC power converters are variable topology systems that display a variety of nonlinear behaviour like bifurcation, quasi-harmonics, and chaos. These nonlinear phenomena affect the normal operation of the converter and force it to change its normal cyclic operation to random behavior (Alturas et al., 2019). There are several reasons why it is important to avoid operating dc-dc power converters in non-linear mode. One of the main reasons is the possibility of having multiple switching in one clock period when the converter works nonlinearly. In addition, when the system is operating in the non-linear region, the AC components for the output voltage and the inductance current increase, which reduces the system's efficiency. Due to these unpredictable and often undesirable phenomena, a focused analysis of the complex dynamic behavior and stability of the dc-dc power converters are required and sometimes compulsory.

System stability is usually characterized by outputs or some of its internal states, if they were growing without limits the system so-called unstable, otherwise it is stable. Not all systems are stable or unstable, as the system could also be marginally stable. The term instability refers to a sudden change of qualitative behaviour of a dynamical system as one or more of its parameters are varied. Mathematically, this is called a bifurcation and the parameter value at which the bifurcation occurs is known as a bifurcation point. Bifurcation diagrams offer a convenient way to investigate the instability by presenting the behaviour of the system graphically. In this diagram, the steady state behaviour of the system is plotted against the bifurcation parameter, and by using this diagram it is possible to visually assessment the steady state behaviour of the system. For example, if the system is operating in stable mode, period-1, for a specific parameters value, the result will be a single point on the diagram,

and if it was period-2 it will be 2 points and so on.

According to the nature of the qualitative change, there are two different types of bifurcation, smooth and non-smooth bifurcation. In smooth one, the bifurcation occurs at the stability boundaries, while in non-smooth kind the bifurcation occurs at the operation boundaries. Consequently, the second kind occurs just in the switched dynamical systems (Elbkosh, 2009; Daho, 2009).

## 1.1 Smooth bifurcation

The bifurcation type can be characterized using the eigenvalues, of the invariant set, movement as a circuit parameter changes. If one or a pair of the eigenvalues goes out of the unit circuit smoothly, then the resulting bifurcation known as a smooth bifurcation. This kind of bifurcation can be analyzed using the jacobian matrix or monodromy matrix. Based on the movement of the eigenvalues as they cross the unit circuit, smooth bifurcation can be divided into three different classes (Elbkosh, 2009; Daho, 2009): 1) Saddle-node bifurcation, which is related to one of the eigenvalues of the jacobian matrix or monodromy matrix becomes equal to +1. Mathematically, one of the eigenvalues goes out of the unit circuit on the positive real line. As a result, the stable fixed-point breaks into two new fixed points and the original fixed point becomes unstable. In discrete-time dynamical system, the saddle-node bifurcation is called fold bifurcation. 2) Period doubling bifurcation, which is relates to one of the eigenvalues becomes equal to -1, the system becomes unstable in period doubling term. Mathematically, one of the eigenvalues goes out of the unit circuit on the negative real line. In this case, the stable fixed point becomes unstable and a new stable double-period appears. In discrete-time dynamical system, this kind of bifurcation is known as a flip bifurcation or sub-harmonic bifurcation. 3) Hopf bifurcation, where a fixed point of a

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dynamical system loses stability as a pair of complex conjugate eigenvalues of the linearization around the fixed point cross the imaginary axis of the complex plane. In discrete-time dynamical system, this kind of bifurcation is known as Neimark bifurcation. All smooth bifurcations are interrupted by non-smooth bifurcation such as border collision which comes into play to disrupt the growth of the standard oscillation.

## 1.2 Non-smooth bifurcation

Non-smooth bifurcation, such as border collision, is discontinuous bifurcation which is characterised by sudden jumps in the eigenvalues. As a result of this bifurcation, the system loses its operation and the behaviour is changed suddenly, e.g. from one orbit to another, from stable to unstable, and so on (Morel et al., 2004). In the literature, many research efforts have been made to study and analyze the dynamic behaviour of dc-dc converters and their stability. The most popular approach is the averaged model which is suitable to predict slow scale instability and lacks to predict fast scale instability (Verghese and Banerjee, 2002; Mazumder, 2001; El Aroudi et al., 2010; El Aroudi et al., 2008). In order to predict fast scale instabilities, there is a need to move to the discrete-time models. Although this approach provides a relatively complete idea about the system behaviour in both slow and fast scale terms, the derivation of the discrete map is very complicated and usually it cannot give a closed form expression for the stability conditions (Verghese and Banerjee, 2002; Mazumder, 2001; El Aroudi et al., 2010). There are many regimes that the switching power converters can work within, such as period-1 operation, period-2 operation, and so on. In order to get the desired periodic operating mode, it is important to choose the appropriate parameters' values (Alturas et al., 2019). These parameters are the switching frequency, the input voltage  $V_{in}$ , the inductance  $L$ , the resonance frequency  $\omega_o$ , the duty cycle  $D$ , the proportional gain  $k_p$ , and the zero frequency  $\omega_z$  of the controller.

Control circuits play an important role in the nonlinear behaviour of DC-DC power converters, to be more precise the pulse with modulation (PWM). The ripple component before the PWM is the main reason that gives birth for fast scale bifurcation. In addition, independently of the parameters changing, if the ripple component before the PWM modulator exceeds a critical value, the system shows fast scale oscillation (El Aroudi et al., 2010; Giaouris et al., 2008; Alarcon et al., 2006; El Aroudi et al., 2006). In this paper, two different approaches are combined together to study and investigate the nonlinear behaviour of DC-DC power converters. The first approach is based on the conventional Routh-Hurwitz (RH) criterion, which is suitable to predict slow scale bifurcation but lacks to predict fast scale bifurcation. The second approach is based on the level of the ripple component at the PWM modulator from the voltage controller, which is used to predict fast scale instability. By combining these two approaches, a complete design-oriented perspective of instability in terms of fast and slow scale has been achieved. The bifurcation diagram for both slow and fast scale bifurcation is obtained using the proportional gain  $k_p$  as a sweep parameter. It has been shown that, the system exhibits nonlinear phenomena such as slow and fast scale oscillation as the value of  $k_p$  is changed. The nonlinearities prediction have been shown analytically and have been confirmed using Matlab simulation.

## 2. THE VOLTAGE MODE CONTROLLED BUCK CONVERTER

Buck converter is the most widely used dc-dc converter topology in power electronics applications due to its simplicity, ease, and precision. Buck converter is used to convert an input dc voltage to a lower level dc voltage with a very high practical efficiency, about 92% (Verghese and Banerjee, 2002). Figure 1 shows a graphical illustration of the buck converter under the PWM voltage mode. The output voltage is sensed and fed back to form the error voltage  $V_{ref} - V_o$  with the help of voltage controller in the form of PI controller. The buck converter has two operation modes. For ON sub-interval when the switch is close, the equations that represent the buck converter are:

$$\frac{dV_C}{dt} = \frac{1}{C} \left( i_L - \frac{V_C}{R} \right) \quad (1)$$

$$\frac{di_L}{dt} = \frac{V_{in} - V_C}{L} \quad (2)$$

During the OFF sub-interval when the switch is open, the equations of the buck converter are:

$$\frac{dV_C}{dt} = \frac{1}{C} \left( i_L - \frac{V_C}{R} \right) \quad (3)$$

$$\frac{di_L}{dt} = \frac{-V_C}{L} \quad (4)$$

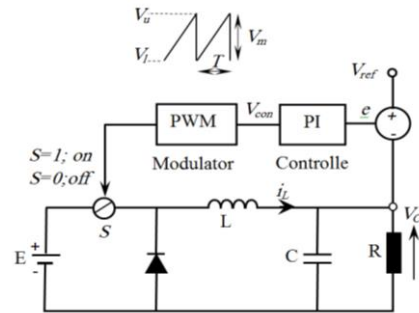


Figure 1: The dc/dc converter topologies.

In continuous conduction mode (CCM), the switch conducts for a fraction  $d$  of each duty cycle and the diode conducts for the remainder,  $1-d$ . Thus, the averaged model will be found by multiplying equations (1) and (2) by  $d$  and equations (3) and (4) by  $1-d$  and then adding them up. By applying the averaged model on the state space equations for this particular system, the following averaged model is obtained:

$$\frac{di_L}{dt} = -\frac{1}{L} V_C + \frac{V_{in}}{L} d \quad (5)$$

where  $d$  is the modulated signal which in the case of PWM is given by:

$$d = \frac{V_{con} - V_1}{V_m} \quad (6)$$

In order to regulate the output voltage to a desired reference voltage, a closed loop in the form of a dynamic PI controller is used which define a third state variable  $e$  (the error):

$$\frac{de}{dt} = V_{ref} - V_C \quad (7)$$

The PI controller is one of the most common controllers that used in control applications. The basic principle of the PI controller is to act upon the variable to be controlled through a combination of two elements:

1) The proportional gain ( $k_p$ ), where the resulting signal is proportional to the error signal. This part is used in order to reduce the rising time and the steady state error but will not get rid of it.

2) The integral part ( $k_i$ ), where the resulting signal is proportional to the integral of the error signal. This part is used to eliminate the steady state error, but it has a bad effect on the transient response. Figure 2 shows the structure of the PI controller and its parts are connected in time domain.

$$V_{con} = k_p \left( e + \frac{k_i}{k_p} \int e dt \right) \quad (8)$$

Having solved equations (5) and (8) and substituting in (6), the duty cycle of this system is given by:

$$d = \frac{k_p \left( V_{ref} - V_C + \omega_z e \right) - V_1}{V_m} \quad (9)$$

where  $V_m = V_U - V_L$  is the amplitude of the ramp signal, and  $\omega_z$  is the zero of the PI controller. By substituting equation (9) in (5), the closed loop averaged model of the buck converter is obtained.

In order to apply the RH criterion, the model in equation (5) has to be transferred to the characteristic polynomial form. There are many approaches that used to transfer the system from its state space representation form to polynomial form. By applying one of these approaches, the characteristic polynomial of this model can be given as follows:

$$\rho_3(s) = a_3(s^3) + a_2(s^2) + a_1(s\lambda + a_0) \quad (10)$$

where:

$$a_3 = 1, a_2 = \frac{1}{RC}, a_1 = \frac{V_m + V_{in}k_p}{LCV_m}, a_0 = \frac{k_p\omega_z V_{in}}{LCV_m} \quad (11)$$

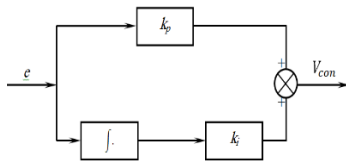


Figure 2: The PI controller diagram

### 3. THE BIFURCATION IN VOLTAGE-MODE CONTROLLED BUCK CONVERTERS

The behaviour of the DC-DC power converters is depending on the circuit parameters. One way to start the phenomena of chaos is the period doubling bifurcation which will continue while there is no stable state variable. The Routh-Hurwitz criterion is a method that used to determine if the roots of the system are stable or not without computing the actual roots. If the system is in polynomial form, so the RH criterion can be applied.

The conventional RH stability criterion has been used in order to examine the slow scale instability index of the buck converter. The RH table is given by (Harwin, 2007):

$$\begin{array}{c|cccc} \lambda^n & a_n & a_{n-2} & a_{n-4} & a_{n-6} & \dots \\ \lambda^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} & \dots \\ \lambda^{n-2} & b_1 & b_2 & b_3 & . & \dots \\ . & c_1 & c_2 & c_3 & . & \dots \\ . & . & . & . & . & \dots \\ \lambda^0 & . & . & . & . & \dots \end{array} \quad (12)$$

A sign change in the first column means the system is unstable. For the polynomial in equation (11) this requires:

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_2} > 0 \quad (13)$$

By substituting  $a_0, a_1, a_2$ , and  $a_3$  from equation (11) in equation (13), the condition for the system to be stable is:

$$\rho_{ss} : \frac{V_{in}k_p}{V_m} (RC\omega_z - 1) < 1 \quad (14)$$

Equation (14) gives a general expression to design a stable system in the slow scale term, which can be used to determine the suitable parameters that ensure slow scale stability. For instance, the critical value of the proportional gain of the PI controller that makes sure the system is stable in slow scale term is given by:

$$k_{p,ss} = \frac{V_m}{V_{in}} \frac{1}{(RC\omega_z - 1)} \quad (15)$$

The system shows fast scale bifurcation depending on the values of the circuit parameters. By increasing the ratio of the converter natural frequency  $f_0$  to the switching frequency  $f_s$ , the converter is more prone to fast scale oscillation (Alarcon et al., 2009; Rodriguez-Vilamitjana et al., 2007; Alarcon et al., 2006). Consequently, it has been presented as a bottom-line hypothesis that the value of the ripple component at the PWM modulator leads to the loss of the period-1 operation in fast scale bifurcation term. As a result, it is possible to use the level of the ripple as an index to predict the fast scale bifurcation in switching devices. In an earlier work, this index has been defined as the ripple at PWM modulator normalized to the ramp signal amplitude and is given by (El Aroudi et al., 2010):

$$\rho : \frac{\Gamma}{V_m} \Delta V_C \quad (16)$$

Where  $\Gamma = k_p \bar{\Gamma}$  is the transfer function of the PI controller evaluated at the switching frequency, and  $\Delta V_C$  is the output voltage ripple? The output voltage ripple is given by:

$$\Delta V_C \approx \frac{V_{in}\omega_0^2 D \bar{D} \pi^2}{2\omega_s^2} \quad (17)$$

Where  $\omega_0 = (LC)^{0.5}$  is the natural frequency,  $D$  is the duty cycle,  $\bar{D} = 1 - D$ , and  $\omega_s = 2\pi f_s$  is the switching frequency. By substituting equation (17) in (16), the index becomes:

$$\rho_{fs} : \frac{k_p \bar{\Gamma} V_{in} \omega_0^2 D \bar{D} \pi^2}{2\omega_s^2 V_m} \quad (18)$$

An approximation form of the critical ripple which as a function of the duty cycle can be given by (El Aroudi et al., 2010):

$$\rho_C : \frac{D \bar{D}}{2(v - 2D \bar{D})} \quad (19)$$

Where  $V$  is given by:

$$v = \frac{2\omega_0^2 \omega_c T^3 D^2 \bar{D}^2 + 4T^2(\omega_c^2 - \omega_0^2) D \bar{D}^2 + 8 - 4T\omega_c}{T^4 \omega_0^4 D^2 \bar{D}^2 + 4T^2(\omega_c^2 - \omega_0^2) D \bar{D}^2 + 8 - 4T\omega_c} \quad (20)$$

As the ripple index exceeds a critical value  $\rho_{critical}$ , the system shows fast scale oscillation. The condition for avoiding losing the fast scale stability of the circuit can be given from:

$$\rho_{fs} < \rho_C \quad (21)$$

From equation (21), the fast scale instability will take place when:

$$\rho_{fs} = \rho_C \quad (22)$$

By substituting equations (18) and (19) in (22), a general expression which can be used to give the critical values of the system parameters that make the system behave in fast scale instability can be given by:

$$\frac{\Gamma V_{in} \omega_0^2 \pi^2}{2\omega_s^2 V_m} - \frac{1}{2(v - 2D \bar{D})} = 0 \quad (23)$$

For example, the critical value of the proportional gain that give birth the fast scale oscillation is:

$$k_{p,fs} = \frac{V_m}{V_{in}} \frac{\omega_s^2}{\bar{\Gamma} \omega_0^2 \pi^2 (v - 2D \bar{D})}, \Gamma = k_p \bar{\Gamma} \quad (24)$$

In order to calculate the values of the slow and fast scale instability indexes of the system, the Matlab M-file has been used. By implementing the

previous equations it has been analytically found that:

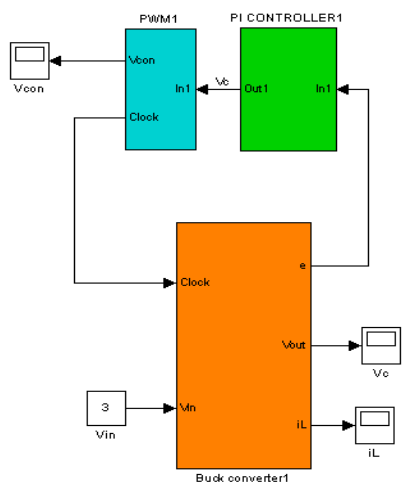
$$k_{p,FS} = 9.9, k_{p,SS} = 0.51 \tag{25}$$

**4. SIMULATION AND RESULTS**

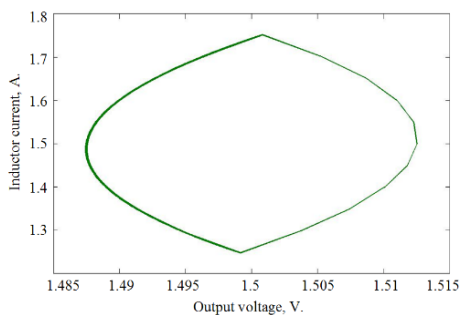
In order to validate the two closed expressions that derived for both the slow and fast scale prediction, the buck converter was implemented in Matlab/Simulink as shown in Figure (3). The switching instants were determined by comparing a ramp signal with the control signal.

The values of the buck converter circuit parameters that used in the simulation are:  $V_{in}=3V, R=1\ \Omega, L=30nH, C=50nF, f_s=50MHz, V_l=0V, V_o=1V, V_{ref}=1.5V$ . The selection of these parameters is based on having a converter with a low ratio of the switching frequency  $f_s$  to the cut-off frequency  $f_c$  of LC filter, hence showing moderately large ripples (El Aroudi et al., 2010). All the previous explained theoretical analyses are programmed using an M-file in Matlab. As the switch is changing its state from ON to OFF periodically by the switching frequency, the circuit shows periodic behaviour. It is clear that, even for a small change in the values of the DC-DC circuit parameters the system may become unstable.

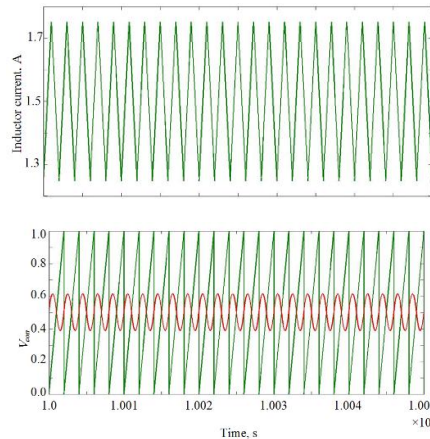
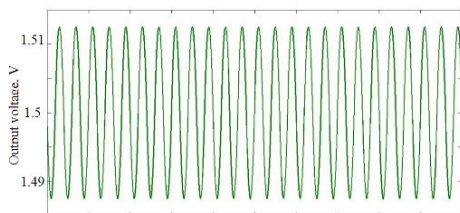
System waveforms as the proportional gain  $k_p=9, \omega_z=14.28\ Mrad/s$  are shown in Figures (4), (5), and (6). It is obvious that the system is working in period-1 operation and hence the system is stable.



**Figure 3:** The block diagram of Matlab-Simulink.



**Figure 4:** Phase portrait of the system,  $k_p=9, \omega_z=14.28Mrad/s$

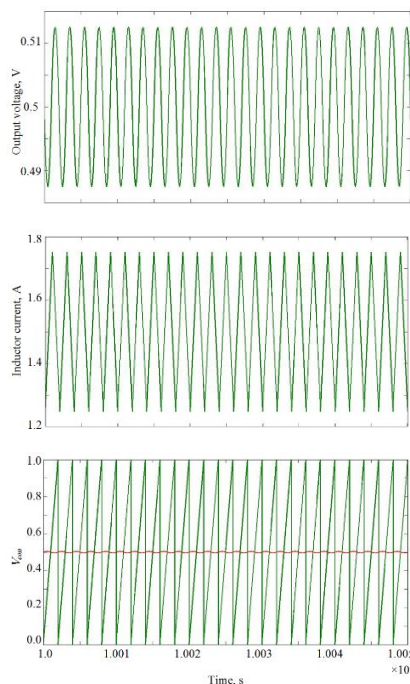


**Figure 5:** Standard period operation before fast scale bifurcation  $k_p=9, \omega_z \approx 14.28Mrad/s$ .

The difference between figure (5) and figure (6) is that the PI controller parameters are chosen differently in each case. In figure (5), the PI controller was chosen with sufficiently high amplitude of the PI controller transfer function so that the ripple magnitude at the PWM modulator will be significantly high. In contrast, in figure (6), the amplitude of the PI controller' transfer function is lower which means the frequency components and the harmonics are smaller in order that the ripple amount at the PWM modulator will be lower.

Under parameters changing, the system may lose the desired periodic oscillation and consequently the stability. As a result, the dynamic behaviour of the buck converter will differ from that of Figures (4), (5), and (6). By considering the proportional gain  $k_p$  of the PI controller as a sweep parameter the system is more prone to introduce bifurcation in the two slow scale and fast scale terms.

System waveforms as the proportional gain  $k_p=10$  and  $\omega_z=14.28Mrad/s$  are illustrated in Figures (7) and (8). It is clear that the system is working in period-2 operation mode which means that the states repeat themselves every two switching cycles. Usually period doubling phenomena end in a chaotic behaviour.



**Figure 6:** Standard period operation before slow scale bifurcation  $k_p=0.3, \omega_z \approx 33Mrad/s$ .

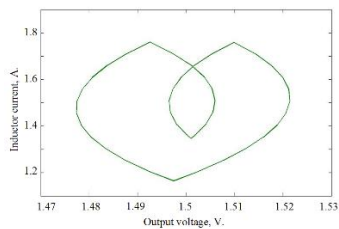


Figure 7: Phase portrait of the system,  $k_p=10$ ,  $\omega_z=14.28\text{Mrad/s}$

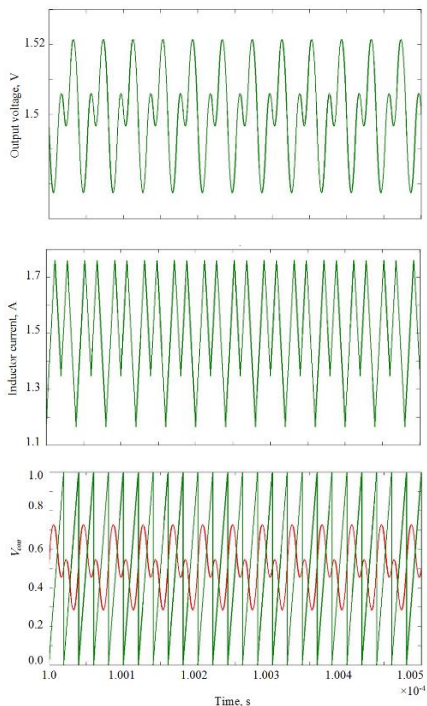


Figure 8: Quasierperiodic oscillation due to a period doubling bifurcation after fast scale bifurcation  $k_p=10$ ,  $\omega_z \approx 14.28\text{Mrad/s}$ .

Another well known nonlinear dynamic behaviour in power electronic circuits is slow scale bifurcation. As the proportional gain has changed to  $k_p = 0.5$  and  $\omega_z = 33 \text{ Mrad/s}$ , the buck converters operates in slow scale oscillation mode as can be seen from figure 9. In Figure (9), the zero frequency  $\omega_z$  is equal to 33 Mrad/s which is greater than  $\omega_c$ , in this case slow scale bifurcation is possible as predicted by equation (14).

In order to validate the previous closed term for both slow and fast scale oscillation the bifurcation diagram has been plotted for two different cases. In Figure (10), the value of the zero frequency  $\omega_z$  is equal to 14.28 Mrad/s which is smaller than  $\omega_c$ , thus and according to equation (23), the system will not show slow scale instability. As a result, period doubling bifurcation takes place at a critical value of the proportional gain  $k_p$  of 9.8 as can be seen from figure (10-a). Figure (10-b) shows evolution of the stability index  $\rho_{FS}$  showing that at the fast scale bifurcation point the stability index  $\rho_{FS}$  is equal to 1 in a good concordance with equation (14).

Figure (11-b) shows the evolution of the stability index  $\rho_{FS}$ , showing that at the slow scale bifurcation point the stability index  $\rho_{FS}$  is equal to the critical ripple  $\rho_c$  in a good concordance with equation (23). As can be seen from Figure (10), the bifurcation point takes place at critical value of the bifurcation parameter  $k_p = 9.8$  in a good concordance with  $k_p = 9.9$  in equation (25). In addition, from Figure (11), it is clear that the bifurcation point takes place when the bifurcation parameter is about 0.51 which is in a good concordance with 0.51 in equation (25). It has been observed that the instability indexes that derived previously for  $k_p$  are depending on the location of the zero of the PI controller  $\omega_z$  with respect to  $\omega_c$ . For example, if  $\omega_z > \omega_c$ , the system will show fast scale bifurcation and slow scale bifurcation is impossible. On the other hand, If  $\omega_z < \omega_c$ , the system will show slow scale bifurcation.

As can be seen from previous figures, the stability analysis of DC-DC buck converter has been investigated using two different methods; the first one is based on the conventional Routh-Hurwitz (RH) criterion which is suitable to predict slow scale instability and cannot predict fast scale oscillation. This shortcoming is because of the elimination of the switching action from the circuit description and averaging the state variables over the switching period. The second approach is based on the use of the ripple component at the PWM modulator to predict sub-harmonic oscillations in DC-DC converters. It has been proved that, the ripple component at the PWM can be used to quantitatively predict fast scale oscillations in DC-DC power converters.

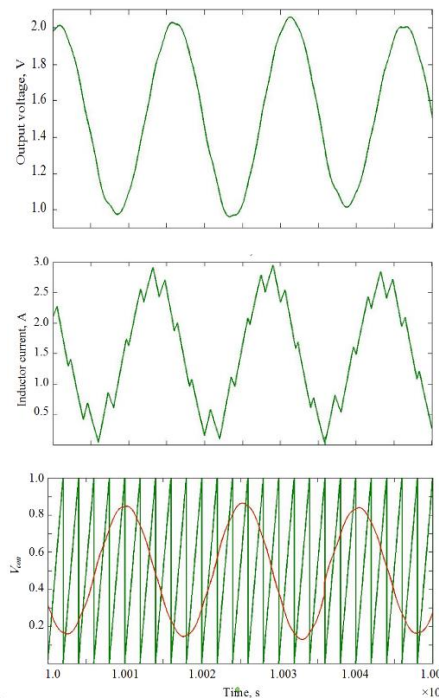
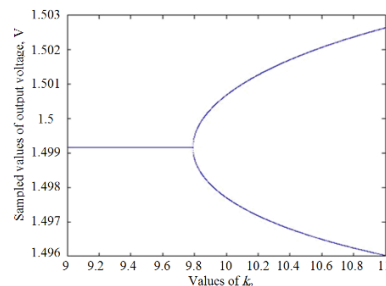
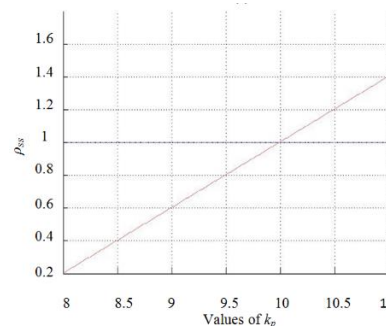


Figure 9: Subharmonic oscillation due to a period doubling bifurcation after fast scale bifurcation  $k_p=0.5$ ,  $\omega_z \approx 33\text{Mrad/s}$ .

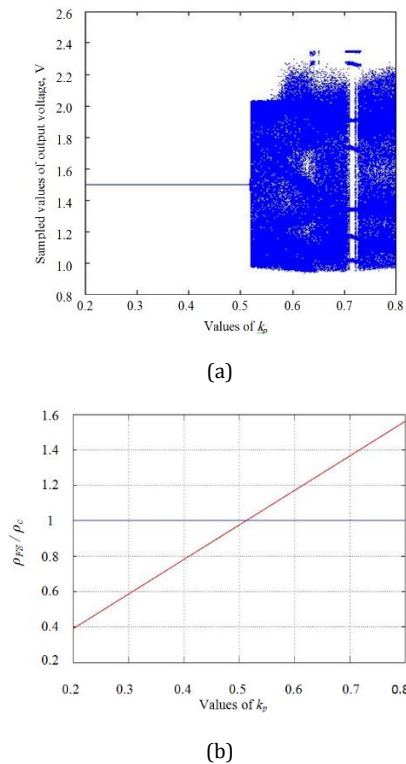


(a)



(b)

Figure 10: (a) Bifurcation diagram of PWM controlled buck,  $\omega_z \approx 14.28 \text{ Mrad} < \omega_c$ . (b) The evolution of fast scale instability index  $\rho_{FS}$ .



**Figure 11:** (a) Bifurcation diagram of PWM controlled buck,  $\omega z \approx 14.28$  Mrab >  $\omega c$ . (b) The evolution of slow scale instability index pss.

## 5. CONCLUSION

In this paper, stability analysis of dc-dc buck converter has been investigated using two different methods; the conventional Routh-Hurwitz (RH) criterion to predict slow scale instability and the use of the ripple component at the PWM modulator to predict sub-harmonic oscillations. It has been proved that, the ripple component at the PWM can be used to quantitatively predict fast scale oscillations in dc-dc power converters. Regardless of the parameter varying, if the ripple component at the PWM modulator exceeds a critical value, the system will show fast scale oscillation. By combining these two approaches it is possible to classify and categorise the design parameter space in different stability areas. Numerical and analytical evidence of validity of this approach in closed-loop buck converter is presented.

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